Factor Theorem

Let's look at a quadratic function to help us.

eg $f(x) = x^2 + 5x + 6$

if we factorise this we get $f(x) = (x+2)(x+3$

which means $(x+2)$ and $(x+3)$ are both factors of the function.

What else do they tell us?

- if we use null factor law, we can find values of x

eg $(x+2)(x+3) = 0$

so $x + 2 = 0$ or $x + 3 = 0$

$x = -2$ $x = -3$

so if we substitute $x = -2$ into $f(x)$ we get

$$f(-2) = \overset{4}{(-2)^2} + \overset{-10}{5 \times -2} + \overset{+6}{6} = 0$$

similarly $f(-3) = \overset{9}{(-3)^2} + \overset{-15}{5 \times -3} + \overset{+6}{6} = 0$

This tells us that if we find a value a, of $P(x)$, such that $P(a) = 0$ then we can write a factor of the polynomial as $(x - a)$

This will help us with higher degrees of polynomials.

eg factorise $x^3 - 3x^2 - 9x - 5$

Where should we start?

think about factors of -5 eg $1, -5, -1, 5$

try $x = 1$ so a factor might be $(x-1)$

$P(1) = 1^3 - 3 \times 1^2 - 9 \times 1 - 5$

$= 1 - 3 - 9 - 5 = -16 \neq 0$ so $(x-1)$ not a factor

factor $(x+1)$

$P(-1) = (-1)^3 - 3 \times (-1)^2 - 9 \times -1 - 5$

$= -1 - 3 \times 1 + 9 - 5$

$= -1 - 3 + 9 - 5 = 0$ $(x+1)$ is a factor

$\therefore$ we know that $(x+1)$ divides exactly into $x^3 - 3x^2 - 9x - 5$ so we could now divide to find other factors (or keep going with factor theorem).

$x+1$	1	-3	-9	-5
	0	1	-4	-5
	1	-4	-5	0

$= x^2 - 4x - 5$

so $(x+1)(x^2 - 4x - 5) = x^3 - 3x^2 - 9x - 5$

factorise this if you can.

$(x+1)(x+1)(x-5)$

so $P(5)$ should $= 0$.

$P(5) = (5)^3 - 3 \times (5)^2 - 9 \times 5 - 5$

$= 125 - 3 \times 25 - 45 - 5$

$= 125 - 75 - 45 - 5 = 0$ ✓